JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 19, No. 3, May–June 1996

Wave Approach for Control of Orientation and Vibration of a Flexible Structure

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A slew maneuver problem of a rigid body equipped with a flexible beam is treated. Control of both the orientation of the rigid body and the vibration of the flexible appendage has been done by using only a single controller input and boundary conditions at the root of the beam. The wave-absorbing control is applied to designing the present control system. Some significant requirements in the design of the controller are proposed to achieve the properness of the compensator transfer functions and the stability of the closed-loop system. The present controller design technique aims to reduce both the reflections of incoming waves and the generations of reflected waves directly excited at the reorientation with the help of only a single controller input. Closed-loop transfer functions are numerically obtained and compared with the open loop. The designed control system is shown to have excellent performance in the frequency region.

Introduction

SPACE structures are usually lightweight because of weight limitations at launch, and they are also likely to be flexible. Vibratory motion of the flexible structures are caused by their reorientation. The vibration suppression is required to assure performance such as precision of the orientation and, in the present case, it is necessary to suppress vibration of the flexible appendage during the slew maneuver. In this paper, a new controller design technique is presented to simultaneously control the orientation and the vibration of the flexible beam by using only a single controller input. The design of the compensator is based on the algebraic analysis of the present system using the traveling-wave approach.

The wave-absorbing control actively changes the vibratory characteristics of flexible structures, such as a structural member of the truss at their joints, and the structural response is described in terms of traveling elastic waves.[1–9] Usually the control is designed to attenuate outgoing waves at the actuator positions so as to minimize the effects of the incoming ones. The traveling wave approach was applied to control the vibration of large space structures (LSS) by von Flotow[2] and von Flotow and Schäfer.[3] They introduced the

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Received April 28, 1994; revision received Aug. 11, 1995; accepted for publication Nov. 10, 1995.

viewpoint that the elastic response of flexible structures may be aptly viewed in terms of their disturbance propagation characteristics. The resulting model depends only on local dynamics and will be robust to errors in modeling the rest of the structure. Such wave-absorbing controllers are used only to suppress the vibrations of the flexible structures. Control of the rigid-body motion, such as the slew maneuver, has not been treated with this controller design methodology. This is because the reorientation of the flexible structures inevitably creates outgoing waves at the actuator positions, whereas conventional wave-absorbing control aims to cancel them. The absence of reports on control of the slew maneuver using the traveling wave approach encouraged us to investigate it.

In this study, the wave-absorbing control is employed to control the slew maneuver of a rigid body with a flexible appendage. Each element of the state vector of the present system can be expressed in terms of wave vectors traveling in the structure. The boundary condition of the flexible appendage and the equation of motion of the central rigid body are expressed in terms of a controller input and the incoming and outgoing waves, which yields the scattering relation at the boundary. The rigid-body motion is taken into account, and so the compensator cannot be designed only by attenuating the outgoing waves at the actuator position, as already mentioned. Outgoing waves are inevitably generated at the actuator positions if the reorientation maneuver is carried out. The compensator is then designed such that some elements of the transfer function matrix from the incoming waves to the sensor output are selected to be the desirable values based on the system analysis and that a multiplier is selected so as to reduce the amplitude of the reflective waves directly excited by the controller input for the orientation. In the present case, the scattering matrix has two rows and columns with only a single controller input, and only two elements of the matrix are freely assignable. This means that the reflected waves are not canceled completely and that the system may be destabilized by the present technique. The instability might be excited through the use of the wave-absorbing control if the closed-loop scattering and generating matrices are designed inadequately. The controlled behavior for the present system is numerically simulated in the frequency region, and the results show that the closed-loop system satisfies the requirements proposed to design the controller. It is also confirmed that the closed-loop system can be stable if the scattering and generating matrices are designed as proposed in the present paper.

System Dynamics

A schematic model of the space structure is shown in Fig. 1. In this figure, the orthogonal axes u and v are rotated around O, the center; θ is an attitude angle of the model, θ_m the desirable attitude angle, and v the displacement from the axis u. The main body is assumed to be fixed in its center of rotation for simplification of the analysis, and it is actuated to rotate in this plane by the torque T. The flexible appendage is subject to the Bernoulli–Euler assumption with one end ($u = l_0$) fixed to the main body and the other end ($u = l_1$) free.

Introducing a variable y,

$$y = v + u\theta \tag{1}$$

the vibrational behavior is described by the following partial differential equation (damping and centrifugal forces are ignored):

$$EI\frac{\partial^4 y}{\partial u^4} + \rho\frac{\partial^2 y}{\partial t^2} = 0 \tag{2}$$

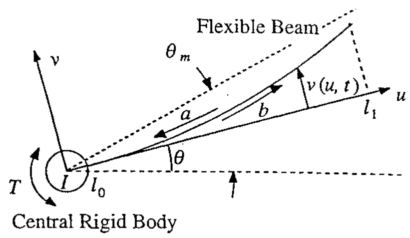

Fig. 1 System model.

together with the boundary conditions

$$v(l_0, t) = 0 \qquad \frac{\partial v}{\partial u}(l_0, t) = 0 \tag{3a}$$

$$\frac{\partial^2 v}{\partial u^2}(l_1, t) = 0 \qquad \frac{\partial^3 v}{\partial u^3}(l_1, t) = 0 \tag{3b}$$

$$y(l_0, t) = l_0\theta \tag{3c}$$

where EI and ρ are the bending rigidity and the mass per unit length of the beam, respectively. The equation of motion for the main body is

$$I\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + M_0 - l_0 Q_0 = T \tag{4a}$$

$$M_0 = M(l_0, t) = EI\frac{\partial^2 v}{\partial u^2}(l_0, t) = 0 \tag{4b}$$

$$Q_0 = Q(l_0, t) = EI\frac{\partial^3 v}{\partial u^3}(l_0, t) = 0 \tag{4c}$$

where I denotes the moment of inertia of the main body, and M and Q are internal bending moment and internal shear force, respectively. The control input T is applied to the main body. Control of reorientation of the beam is to change the dynamic state of the system from an initial one with $\theta = 0$ and $\mathrm{d}\theta/\mathrm{d}t = 0$ to a desirable one with $\theta = \theta_m$ and $\mathrm{d}\theta/\mathrm{d}t = 0$.

Equation (2) is Laplace transformed into an ordinary differential equation, and the analysis proceeds as in Ref. 1. The waves a_1 and a_2 travel in the negative u direction and b_1 and b_2 in the positive u direction. The relation between the waves at two points $u = u_0$ and $u = u_1$ ($u_1 \geq u_0$) on the beam is obtained with the help of the general solutions of Eq. (2) in the complex Laplace plane as follows:

$$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}_{u=u_1} = e^{qh}\begin{bmatrix} \cos qh & -\sin qh \\ \sin qh & \cos qh \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}_{u=u_0} \tag{5a}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}_{u=u_1} = e^{-qh}\begin{bmatrix} \cos qh & \sin qh \\ -\sin qh & \cos qh \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}_{u=u_0} \tag{5b}$$

where $h = u_1 - u_0$ and $q = (s/2)^{1/2}(\rho/EI)^{1/4}$. It is seen from Eqs. (5) that there is no interference between the waves traveling into the directions opposite to each other and that the transfer function from the upstream to the downstream of each wave is causal. The relation between the state vector $\boldsymbol{Y} = [y \;\; \mathrm{d}y/\mathrm{d}u \;\; M \;\; Q]$ and the wave vector $\boldsymbol{W} = [a_1 \;\; a_2 \;\; b_1 \;\; b_2]$ is

$$\boldsymbol{Y} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ q & -q & -q & q \\ 0 & -X_0 & 0 & -X_0 \\ -X_1 & -X_1 & X_1 & X_1 \end{bmatrix}\boldsymbol{W} \tag{6}$$

where $X_0 = 2q^2 EI$ and $X_1 = qX_0$.

The boundary condition at the root of the beam [Eq. (3c)] and the equation of motion for the main body [Eq. (4a)] are written in terms of the state vector $\boldsymbol{Y}$ at $u = l_0$ in matrix form,

$$\begin{bmatrix} 0 & Is^2 & -1 & l_0 \\ 1 & -l_0 & 0 & 0 \end{bmatrix} Y(l_0) = \begin{bmatrix} T \\ 0 \end{bmatrix} \tag{7}$$

which can be rewritten in terms of the wave vector using Eq. (6) as follows:

$$UW(l_0) = \begin{bmatrix} T \\ 0 \end{bmatrix} \tag{8a}$$

$$U^T = \begin{bmatrix} V_0 q V_1 & 1 - V_2 \\ V_0(\rho - \rho V_2 - V_3) & V_2 \\ -V_0 q V_1 & 1 + V_2 \\ V_0(\rho + \rho V_2 + V_3) & -V_2 \end{bmatrix} \tag{8b}$$

where $V_0 = 2EIq^2/\rho$, $V_1 = 2Iq^2 - l_0\rho$, $V_2 = l_0 q$, and $V_3 = 2EIq^3$. Partitioning W into the incoming waves W_{in} and the outgoing waves W_{out}, Eqs. (8) are rewritten as

$$W_{\text{out}} = SW_{\text{in}} + BT \tag{9a}$$

where

$$W_{\text{in}} = [a_1 \quad a_2] \qquad W_{\text{out}} = [b_1 \quad b_2] \tag{9b}$$

and

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} P_1 - P_2 - \rho & -P_0 \\ P_0 - 2P_2 & P_1 + P_2 - \rho \end{bmatrix} \tag{9c}$$

with $P_0 = 2l_0\rho q$, $P_1 = l_0 q P_0$, $P_2 = 2Iq^3$, and $\Delta = \rho + P_0 + P_1 + P_2$. The matrix S is the transfer function matrix from the incoming waves to the reflected waves, and it is called the open-loop scattering matrix; the matrix B is the transfer function matrix from the control input to the reflective waves and it is called the open-loop generating matrix:

$$B = \begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} l_0\rho \\ \rho(1 + l_0 q) \end{bmatrix} \tag{10}$$

Note from Eq. (9a) that the outgoing waves W_{out} are produced by the reflection of the incoming waves W_{in} and are generated by the control input T. For details of the preceding derivation, refer to Ref. 1.

Wave-Absorbing Control

In the present case, the orientation and the vibration of the flexible beam are controlled by only a controller input. The controller input T is sum of the two inputs, that is, T_A for the orientation control of the beam and T_W for the vibrations suppression using the wave-absorbing control:

$$T = T_A + T_W \tag{11}$$

The wave-absorbing controller is designed using Eqs. (8), and it is limited to the form

$$T_W = Cz = C[FW_{\text{in}} + GW_{\text{out}}] \tag{12}$$

where $C \in \mathrm{C}^{1\times N}$; $z \in \mathrm{C}^{N\times 1}$; $F, G \in \mathrm{C}^{N\times 2}$; N is the number of the sensor outputs; and the controller input T_W is in output feedback form. On making use of Eq. (6), the sensor outputs z is expressed by the wave vectors as in Eq. (12), and the closed-loop scattering behavior at the root of the beam is obtained from Eqs. (10–12) as follows:

$$W_{\text{out}} = S_{\text{cl}} W_{\text{in}} + B_{\text{cl}} T_A \tag{13}$$

where S_{cl} is the closed-loop scattering matrix

$$S_{\text{cl}} = [I - BCG]^{-1}[S + BCF] = S + S_a(C) \tag{14}$$

If only a single controller input is available, the matrix S_a is written as

$$S_a = k_B BC[F + GS] \tag{15}$$

where $k_B = 1/(1 - CGB)$, and then the matrix B_{cl} is

$$B_{\text{cl}} = [I - BCG]^{-1}B = k_B B \tag{16}$$

Note that if the rigid-body motion is not treated in the controller design, the closed-loop generating matrix does not exist.

Now let us consider two cases for design of the compensator. One is to derive the compensator through the use of the closed-loop scattering matrix only and the other through the use of both the scattering matrix and the generating matrix.

Case 1

The compensator is derived through the use of only the closed-loop scattering matrix. In the wave-based techniques, the compensator is usually designed by setting some elements of the closed-loop scattering matrix zeros or desirable values so as to reduce the effect of the incoming waves on the reflected ones. If the desirable scattering matrix is defined as S_{rf}, the compensator can be solved by making use of Eq. (14) in the matrix form

$$C^T = H/(DH - 1) \tag{17a}$$

where

$$D = B^T G^T \tag{17b}$$

$$H = [B \otimes [F + GS]^T]^{-1}\text{vec}\big[S^T - S_{\text{rf}}^T\big] \tag{17c}$$

and vec$A$ is a column string of A, and $A \otimes B$ is a Kronecker product of A and B (see Appendix).

Some theoretical requirements for design of a compensator are discussed in the following paragraphs.

1) In the present case, the scattering matrix has two columns and rows and only the single controller input is available. It will be stated in case 2 that the rank of S_a is unity and that the number of assignable elements of the scattering matrix is less than or equal to two. Thus the reflective waves cannot be canceled completely at the root of the beam. If two control inputs are available for us, each element of the scattering matrix is freely assignable, which leads to no reflection at the root. The implementation of two controller inputs might require complicated mechanisms for the actuators. The desirable compensator can be obtained in matrix form as shown in Eqs. (17), and it can be analyzed easily. If only a single controller input is employed, Eq. (17) cannot be used because the matrix $B \otimes [F + GS]^T$ in Eq. (17c) is singular. It is verified that equations similar to Eqs. (17) can be easily formulated for the cases with two or less elements in the scattering matrix.

2) It is desirable at implementation that the compensator be of finite magnitude over the entire frequency region, that is, a proper rational function with respect to the Laplace variable. If all of the compensators are proper, each element of vec$(S - S_{rf})$ in Eqs. (17) must be strictly proper and its relative order should be larger than a certain number that depends on the number of the available sensor outputs. Note that the relative order is the difference between the orders of the denominator and the numerator in a rational function. It is shown in Eq. (9c) that all of the elements except for the $(1, 2)$ element are of the same order for their numerators in the open-loop scattering matrix. If each element of S_{rf} is simply designed to be proper, e.g., to be zeros except for the $(1, 2)$ element, the relative order of the matrix vec$(S - S_{rf})$ is determined by those of the open-loop scattering matrix, which leads to be an improper compensator. These constraints exist because the rigid-body motion is included in the present system. In the wave-absorbing control, a dominant part of the elements of the scattering matrix are set to zero in many cases if all of the reflective waves cannot be canceled. If the $(1, 2)$ element of the scattering matrix is set to zero, the compensators become proper and do not yield well to damping performance. It is because the $(1, 2)$ element of the scattering matrix does not directly affect the performance of the bending moment at the root of the beam M_0:

$$M_0 = -2EIq^2[S_{\text{cl}21} \quad S_{\text{cl}22} + 1]W_{\text{in}} + \big[0 \quad -2EIq^2\big]B_{\text{cl}}T_A \tag{18}$$

3) Successful performance cannot be achieved if the compensators are designed only by reducing the reflected waves without taking into account the rigid-body motion. The angle θ and the bending moment M_0 are expressed by making use of the incoming waves and the control input at the root of the beam in the form

$$\begin{bmatrix} \theta \\ M_0 \end{bmatrix} = \left[\begin{bmatrix} F_\theta \\ F_{M_0} \end{bmatrix} + \begin{bmatrix} G_\theta \\ G_{M_0} \end{bmatrix} S_{\text{cl}}\right] W_{\text{in}} + \begin{bmatrix} G_\theta \\ G_{M_0} \end{bmatrix} B_{\text{cl}} T_A \tag{19a}$$

where

$$F_\theta = [q \quad -q] \qquad G_\theta = [-q \quad q] \tag{19b}$$

$$F_{M_0} = G_{M_0} = \big[0 \quad -2EIq^2\big] \tag{19c}$$

If the coefficient matrix of the incoming waves is set to be a null matrix, the closed-loop scattering matrix is determined by

$$\begin{bmatrix} F_\theta \\ F_{M_0} \end{bmatrix} + \begin{bmatrix} G_\theta \\ G_{M_0} \end{bmatrix} S_{\text{cl}} = 0 \tag{20a}$$

Equation (20a) yields as the solution

$$S_{\text{cl}} = -I \tag{20b}$$

A desirable selection on the scattering matrix is now described in Eq. (20b), but all of the elements cannot be selected to be completely desirable over the entire frequency region. The open-loop scattering matrix has the following characteristic:

$$\lim_{s \to 0} S = -I \tag{21}$$

The open-loop scattering matrix approximately satisfies Eq. (20b) for a low-frequency region, and so the selection on the scattering matrix as in Eq. (20b) seems natural. Taking into account this characteristic of the open-loop scattering matrix, the matrix S_a is designed to satisfy the following condition:

$$\lim_{s \to 0} S_a = 0 \tag{22}$$

In summary, only the scattering matrix is modified by the output feedback and selected to be a desirable one in the present case. As a result all of the compensators can be proper, and the effect of the incoming waves on the sensor outputs is reduced. It is confirmed numerically that the stability of the closed-loop system is deteriorated in many cases because the generating matrix is not taken into consideration in the present case.

Case 2

Both the scattering and generating matrices are taken into account in the controller design technique. Let the first and second rows of S_a be S_{ar1} and S_{ar2}, respectively, which yields the relation

$$S_{ar2} = (B_{21}/B_{11}) S_{ar1} \tag{23}$$

It is evident from Eq. (23) that only two compensators can be determined by selecting the closed-loop scattering matrix adequately, that is, there are only two independent equations in the present case. It is necessary to take into account the generating matrix so as to guarantee the stability of the closed-loop system as stated in case 1. The shaping of the closed-loop generating matrix is because of the selection on the multiplier k_B as shown in Eq. (16). Combination of Eq. (16) and part of Eq. (17) gives as the solution

$$C^T = \begin{bmatrix} B \otimes [F + GS] - \text{vec}\left(S^T - S_{rf}^T\right) B^T G^T \\ B^T G^T \end{bmatrix}^{-1} \times \begin{bmatrix} \text{vec}\left(S^T - S_{rf}^T\right) \\ 1/k_B - 1 \end{bmatrix} \tag{24}$$

where the first parenthesis on the right-hand side must be nonsingular by adequately selecting the number of the assignable elements of the scattering matrix. Therefore, Eq. (23) is not attainable by using all of the elements of the scattering matrix as in case 1. If $s\theta$ and $l_0 Q_0 - M_0$ are selected as the sensor outputs, only a single element of the scattering matrix and the multiplier k_B are selected to be a desirable value. Note that the closed-loop stability is guaranteed by making use of the negative constant compensators with the present sensor outputs.[10] In this case, the controller input T_W is written as

$$\begin{aligned} T_W &= C_1 s\theta + C_2(M_0 - l_0 Q_0) \\ &= [C_1 \quad C_2][F \quad G][W_{\text{in}} \quad W_{\text{out}}]^T \end{aligned} \tag{25a}$$

where

$$F = \begin{bmatrix} sq & -sq \\ X_0 q l_0 & -X_0(1 - l_0 q) \end{bmatrix}, \quad G = \begin{bmatrix} -sq & sq \\ -X_0 q l_0 & X_0(1 + l_0 q) \end{bmatrix} \tag{25b}$$

Equations (24) and (25) give as the designed compensators

$$C_1 = IsA_0 - \frac{1 + 2l_0 q + 2l_0^2 q^2}{qA_1} A_2 \sqrt{EI/\rho} \tag{26a}$$

$$C_2 = -A_0 - (A_2/\rho A_1) \tag{26b}$$

where $A_0 = 1 - 1/k_B$, $A_1 = 2(1 + l_0 q)$, and $A_2 = \Delta S_{a22}/k_B$. Direct substitution of k_B as an nth-order rational function into Eqs. (26) and a straightforward but elaborate manipulation yield as a sufficient condition for the compensators to be proper:

$$k_B = \frac{q^n + N_0 q^{n-1} + N_1 q^{n-2} + \cdots + N_{n-1}}{q^n + N_0 q^{n-1} + D_1 q^{n-2} + \cdots + D_{n-1}} \tag{27}$$

where $n \geq 2$ and k_B should be less than unity in magnitude so as to reduce the reflective waves generated by the controller input T_A as shown in Eq. (16).

Numerical Simulation

The closed-loop responses are analyzed by making use of numerical simulations. The physical parameters for the present system are shown in Table 1. The (2, 2) element of the closed-scattering matrix and the multiplier k_B are selected to be a desirable value as shown in case 2:

$$S_{\text{cl}22} = S_{22} + S_{a22} \tag{28}$$

$$S_{a22} = -\frac{T_1 s}{(1 + T_1 s)(1 + T_2 s)(1 + \sqrt{T_3 s})^3} S_{22} \tag{29}$$

$$k_B = \frac{p(1 + \sqrt{T_4 s})^2}{1 + 2p\sqrt{T_4 s} + pT_4 s} \tag{30}$$

$$T_A = K(\theta_m - \theta) \tag{31}$$

where the values of T_i, p, and K are shown in Table 2 and they are tuned to achieve the stability and the good maneuvering performance in the closed-loop systems as well as to satisfy the requirements as stated in case 2. Equation (29) is almost a product of the first-order systems, and k_B is in the simplest form of Eq. (27). All of the transfer functions of the present system are calculated by making use of exact solutions of the Laplace-transformed equation of motion with viscous damping $d_1 \partial v/\partial t$ and structural hysteretic damping $jd_2 \partial^4 v/\partial u^4$. The damping coefficients d_1 and d_2 are shown in Table 2.

Table 1 Model parameters for the numerical simulations

Flexible beam		
Mass per unit length, kg/m	ρ	0.11
Length, m	$L = l_1 - l_0$	1.20
Bending rigidity, Nm^2	EI	1.76×10^{-1}
Central rigid body		
Radius, m	l_0	3.38×10^{-2}
Moment of inertia, kgm^2	Ir	3.43×10^{-2}

Table 2 System parameters for the numerical simulations

Scattering matrix	
T_1	1.59
T_2	6.37×10^{-2}
T_3	5.31×10^{-2}
Generating matrix	
T_4	7.96×10^{-2}
p	0.15
Feedback gain	
K	0.20
Damping coefficients	
d_1	5.0×10^{-4}
d_2	5.0×10^{-4}

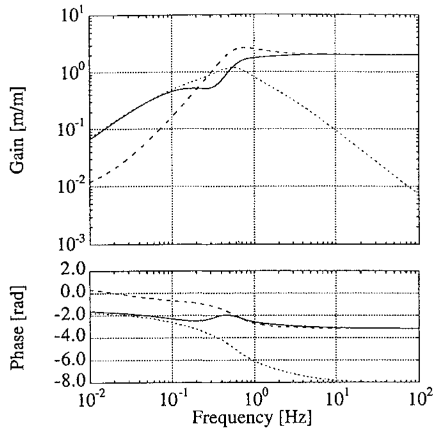

Fig. 2 Open-loop (broken line) and closed-loop (solid line) transfer functions of the (2, 1) element of the scattering matrix and the difference between both cases (dotted line).

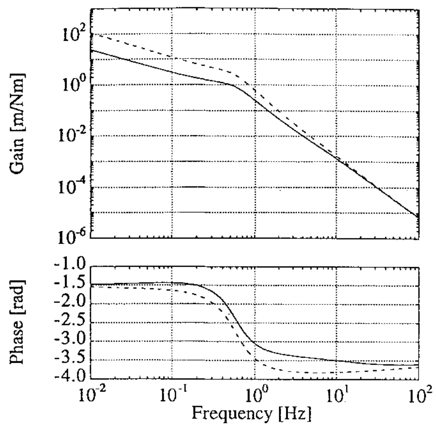

Fig. 3 Open-loop (broken line) and closed-loop (solid line) transfer functions of the (2, 1) element of the generating matrix.

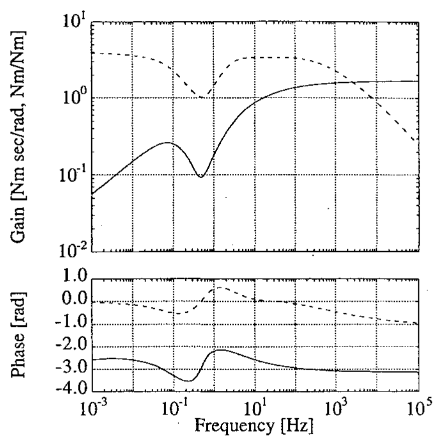

Fig. 4 Transfer functions of the designed compensators; C_1, solid line and C_2, broken line.

Figure 2 shows the typical frequency responses of the (2, 1) element of the open- and closed-loop scattering matrices with the difference between the two cases and Fig. 3 shows the transfer functions of the (2, 1) element of the generating matrices. The closed-loop scattering and generating matrices are shown to satisfy the requirements for the design of the controller as stated in case 2. The transfer functions of the designed compensators are shown in Fig. 4. It can be easily seen that the compensators are proper rational functions. Figure 5 compares the open- and closed-loop transfer functions from the controller input T_A to the attitude angle θ, Fig. 6 the transfer functions from T_A to the bending moment M_0 at the root, and Fig. 7 the transfer functions from the command angle θ_m to the attitude angle θ. Figures 5–7 show that the wave-absorbing control successfully achieves vibration suppression in the low-frequency region. In the present technique, the stability of the system is not guaranteed because of the selection of the design parameters. It would be difficult, however, to achieve the closed-loop stability through only the use of shaping the scattering matrix in the frequency domain, as shown in case 1. This is because the reflective waves are inevitably generated by the controller input T_A for the reorientation of the flexible structure.

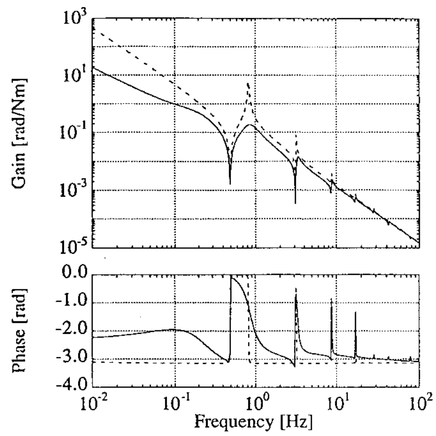

Fig. 5 Open-loop (broken line) and closed-loop (solid line) transfer function from T_A to θ.

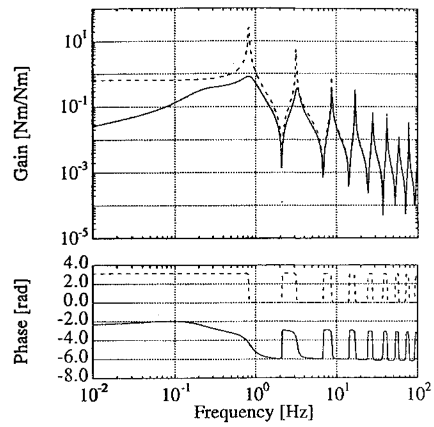

Fig. 6 Open-loop (broken line) and closed-loop (solid line) transfer function from T_A to M_0.

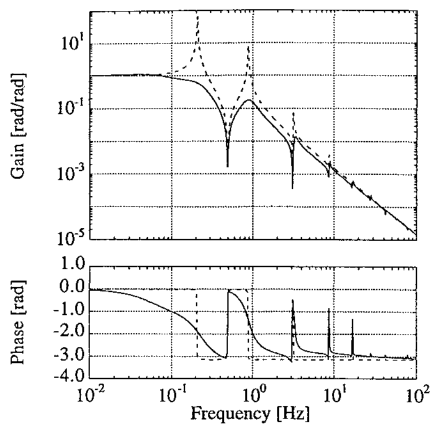

Fig. 7 Open-loop (broken line) and closed-loop (solid line) transfer function from θ to θ_m.

Conclusion

This paper studies both reorientation and vibration suppression of a flexible structure analytically and numerically. The vibratory motion is excited during the slew maneuver of a rigid body equipped with a flexible appendage, and the behavior is described in terms of the elastic waves modes traveling in directions opposite each other. In the present system only a single control input is available, the torque into the central rigid body. The compensator is designed with the help of only the boundary conditions at the root of the beam. The wave-absorbing control is applied for designing the control system for the present case. The aim of the desirable compensator is to reduce the effect of the incoming waves on the controlled variables and to reduce the waves generated directly by the control input for the reorientation, and it can be analytically solved in the matrix form. It is verified from the system analysis that the present technique cannot be employed in the system with more than two sensor outputs to be fedback into only the single controller input. All of the elements of the scattering matrix can be assignable if more than two controller inputs are available, which leads to the perfect cancellation of the reflective waves. Some requirements are given for the closed-loop scattering and generating matrices to guarantee the properness of the compensator and the stability, as well as good performance of the closed-loop system. The closed-loop transfer functions are numerically analyzed in the frequency domain, and they show that the present controller design technique is able to control the slew maneuver of the flexible structure successfully.

Appendix: Matrix Operator

The definition of the Kronecker product and the column string for any matrices $A = (a_{ij}) \in C^{m \times n}$ and $B \in C^{p \times q}$ are as follows.

Kronecker product:

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1n}B \\ a_{21}B & a_{22}B & \cdots & a_{2n}B \\ \vdots & \vdots & & \vdots \\ a_{n1}B & a_{n2}B & \cdots & a_{nn}B \end{bmatrix} \tag{A1}$$

Column string:

$$\text{vec}(A) = [a_{11} \ \cdots \ a_{m1} \ \ a_{12} \ \cdots \ a_{m2} \ \ a_{1n} \ \cdots \ a_{mn}] \tag{A2}$$

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